A simple angle integration method for the determination of capture reaction cross sections

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Outline

• Angle distribution method
• Angle integration method
• Future plans and conclusions
Angular distribution method

Capture reactions

For every beam energy and detection angle $\rightarrow$ $\gamma$-ray energy spectrum
Cross section determination

\[ \sigma_T = \frac{A}{N_A \xi} Y \]

- \( A \): atomic weight
- \( N_A \): Avogadro number
- \( \xi \): target thickness
- \( Y \): reaction yield \( \rightarrow Y = \sum_{i=1}^{N} (A_0)_i \)

Calculation of \((A_0)_i\):
- Spectrum analysis \( \rightarrow I(E_i^\gamma, E_{j\text{ beam}}, \theta_\kappa) \)
- Corrections \( \omega(\theta), \varepsilon_{\text{abs}}(\theta) \) \( \rightarrow I'(E_i^\gamma, E_{j\text{ beam}}, \theta_\kappa) \)
- Normalization \( Q \) \( \rightarrow Y(E_i^\gamma, E_{j\text{ beam}}, \theta_\kappa) \)
- Fit \( Y(\theta) \) with function:
  \[ W(\theta) = A_0 (1 + \sum_k \alpha_k P_k(\cos \theta)), \quad k = 2, 4, \ldots \]
**Step 1:** Experimental determination of the differential gamma-production cross section

\[ \frac{d\sigma_{\gamma}}{d\Omega} (E^\gamma_i, E^{\text{beam}}_j, \theta_\kappa) \]

Angle integration method*

- **Step 1:** Experimental determination of the differential gamma-production cross section
  \[ \frac{d\sigma_\gamma}{d\Omega} (E_i^\gamma, E_j^{beam}, \theta_\kappa) \]

- **Step 2:** Determination of the angle-integrated gamma-ray production cross section
  \[ \sigma_\gamma (E_i^\gamma, E_j^{beam}) \]

Angle integration method

- **Step 1**: Experimental determination of the differential gamma-production cross section

\[ \frac{d\sigma_\gamma}{d\Omega} \left( E_\gamma^i, E_{j \text{beam}}, \theta_\kappa \right) \]

- **Step 2**: Determination of the angle-integrated gamma-ray production cross section

\[ \sigma_\gamma \left( E_\gamma^i, E_{j \text{beam}} \right) \]

- **Step 3**: Determination of the reaction cross section for every beam energy

\[ \sigma_T \left( E_{j \text{beam}} \right) \]

Angle integration

Step 2:

Differential gamma-production cross section:

\[
\frac{d\sigma_\gamma}{d\Omega}(\theta_\gamma) = \frac{\sigma_\gamma}{4\pi} \sum_k c_k P_k(\cos\theta_\gamma), \quad k = 0, 2, 4, ...
\]

Numerical integration of the differential cross section by the Gaussian quadrature method:

\[
\sigma_\gamma = 2\pi \int_{-1}^{1} \frac{d\sigma_\gamma}{d\Omega}(x)dx = 2\pi \sum_{i=1}^{n} w_i \frac{d\sigma_\gamma}{d\Omega}(x_i)
\]

\[x = \cos\theta, \quad n: \text{number of detectors}\]

Requirement: The sum gives the exact result for the integral → Calculation of the appropriate \(w_i, x_i\)
Determination of $w_i, x_i$

- $x_i = \cos \theta_i$ equals to the root of the Legendre polynomial $P_{2n}$
- The weight equals to:

<table>
<thead>
<tr>
<th>Number of detectors</th>
<th>Weight $w_i$</th>
<th>Angle $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
<td>2</td>
<td>55</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>1.304</td>
<td>31</td>
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<td>70</td>
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<td></td>
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<td>49</td>
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<tr>
<td></td>
<td>0.343</td>
<td>21</td>
</tr>
</tbody>
</table>

Case of 2 detectors:

$$\begin{cases} 
    w_1 + w_2 = 2 \\
    w_1 P_2(x_1) + w_2 P_2(x_2) = 0 \\
    w_1 P_4(x_1) + w_2 P_4(x_2) = 0 
\end{cases}$$

Exact approach for polynomials of degree: $\leq 4n - 2$
Determination of $w_i, x_i$

- $x_i = \cos \theta_i$ equals to the root of the Legendre polynomial $P_{2n}$
- The weight equals to:

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<th>Angle $\theta$</th>
</tr>
</thead>
<tbody>
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<td>104</td>
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<tr>
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<td>0.722</td>
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<td>131</td>
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<td></td>
<td>0.343</td>
<td>21</td>
</tr>
<tr>
<td></td>
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<td>159</td>
</tr>
</tbody>
</table>

Case of 2 detectors:

$110^\circ$ (or $70^\circ$) and $150^\circ$ (or $30^\circ$) $\rightarrow$ $P_4(\cos \theta) = 0$

$$\sigma_\gamma = 2\pi \left[ w_1 \frac{d\sigma}{d\Omega} (110^\circ, E_{beam}) + w_2 \frac{d\sigma}{d\Omega} (150^\circ, E_{beam}) \right]$$
Step 3:

The reaction cross section for each beam energy is calculated using the formula:

$$
\sigma(E) = \sum_{i=1}^{N} \sigma_{\gamma}(E^\gamma, L_i \rightarrow g.s.)
$$

$N$: number of transitions that lead to the ground state

$\sigma_{\gamma}$: gamma-ray production cross-section for gamma-ray energy $E^\gamma$
Measurements by the angle integration method

Good agreement of the data with existing experimental data and with TALYS calculations

Future plans

Application of angle integration method to well known reaction

\[ ^{92}\text{Mo}(p, \gamma)^{93}\text{Tc} \]

Cross section measurement of proton capture reaction

\[ ^{66}\text{Zn}(p, \gamma)^{67}\text{Ga} \]
Conclusions

- Need for fewer detectors
- Less experimental data to be analysed
- Validation of the angle integration method for the calculation of capture reaction cross section
Thank you for your attention
<table>
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<tr>
<th>Number of detectors</th>
<th>$P_{2n} = 0$</th>
<th>Weight $w_i$</th>
<th>Angle $\theta$</th>
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<td>0.93247</td>
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<td>21.18</td>
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Solution of the equation system

For $k = 2$

\[ \int_{-1}^{1} \frac{d\sigma}{d\Omega} (\cos\theta) d\cos\theta = \sum_{i=1}^{n} w_i \frac{d\sigma}{d\Omega} (\cos\theta_i) \]

\[ \Gamma \alpha n = 2: \]

- \[ \frac{d\sigma}{d\Omega} (\cos\theta_i) = 1 + c_2 P_2(\cos\theta_i) + c_4 P_4(\cos\theta_i) \]
- \[ \int_{-1}^{1} \frac{d\sigma}{d\Omega} (\cos\theta) d\cos\theta = w_1 \frac{d\sigma}{d\Omega} (\cos\theta_1) + w_2 \frac{d\sigma}{d\Omega} (\cos\theta_2) \]
- \[ \int_{-1}^{1} \frac{d\sigma}{d\Omega} (\cos\theta) d\cos\theta = \int_{-1}^{1} \{1 + c_2 P_2(\cos\theta_i) + c_4 P_4(\cos\theta_i)\} d\cos\theta = 2 + 0 + 0 \]

\[ w_1 (1 + c_2 P_2(\cos\theta_1) + c_4 P_4(\cos\theta_1)) + w_2 (1 + c_2 P_2(\cos\theta_2) + c_4 P_4(\cos\theta_2)) = 2 \Rightarrow \]

\[ \begin{cases} w_1 + w_2 = 2 \\ w_1 P_2(\cos\theta_1) + w_2 P_2(\cos\theta_2) = 0 \\ w_1 P_4(\cos\theta_1) + w_2 P_4(\cos\theta_2) = 0 \end{cases} \]