A simple angle integration method for the determination of capture reaction cross sections

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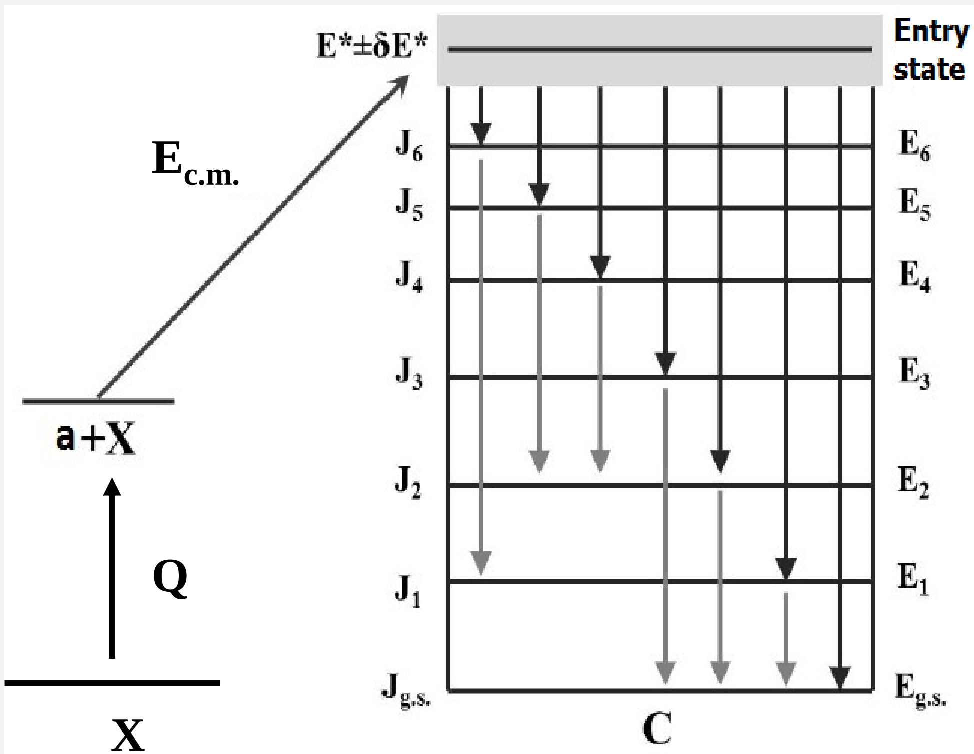
25th Symposium of the Hellenic Nuclear Physics Society,
NCSR “Demokritos”, June 3-4, 2016

Outline

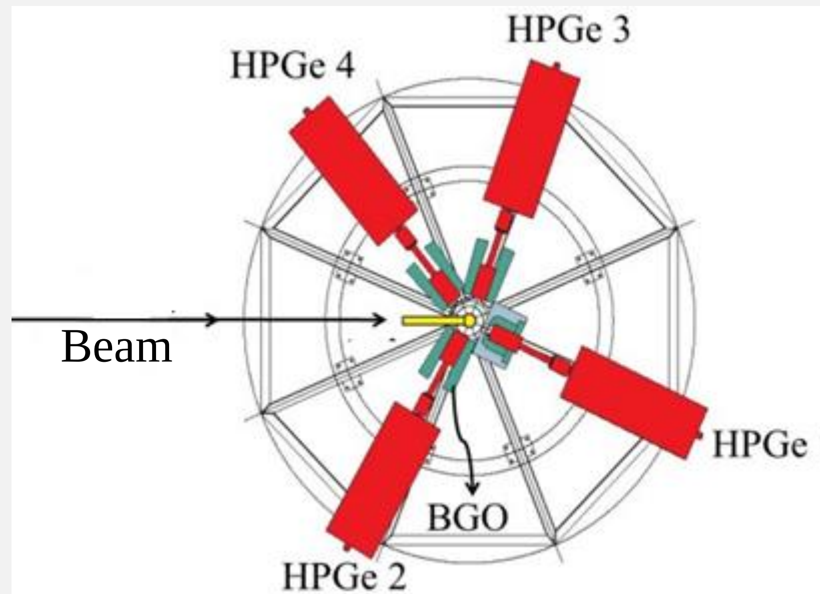
- Angle distribution method
- Angle integration method
- Future plans and conclusions

Angular distribution method

Capture reactions



Experimental setup



For every beam energy and detection angle → γ-ray energy spectrum

Cross section determination

$$\boldsymbol{\sigma}_{\mathbf{T}} = \frac{\mathbf{A}}{\mathbf{N}_{\mathbf{A}}\,\boldsymbol{\xi}}\boldsymbol{Y}$$

A: atomic weight
$\boldsymbol{N_A}$: Avogadro number
$\boldsymbol{\xi}$: target thickness
$\boldsymbol{Y}$: reaction yield $\rightarrow \Upsilon = \sum_{i=1}^{N} (A_0)_i$

Calculation of $(A_0)_i$:

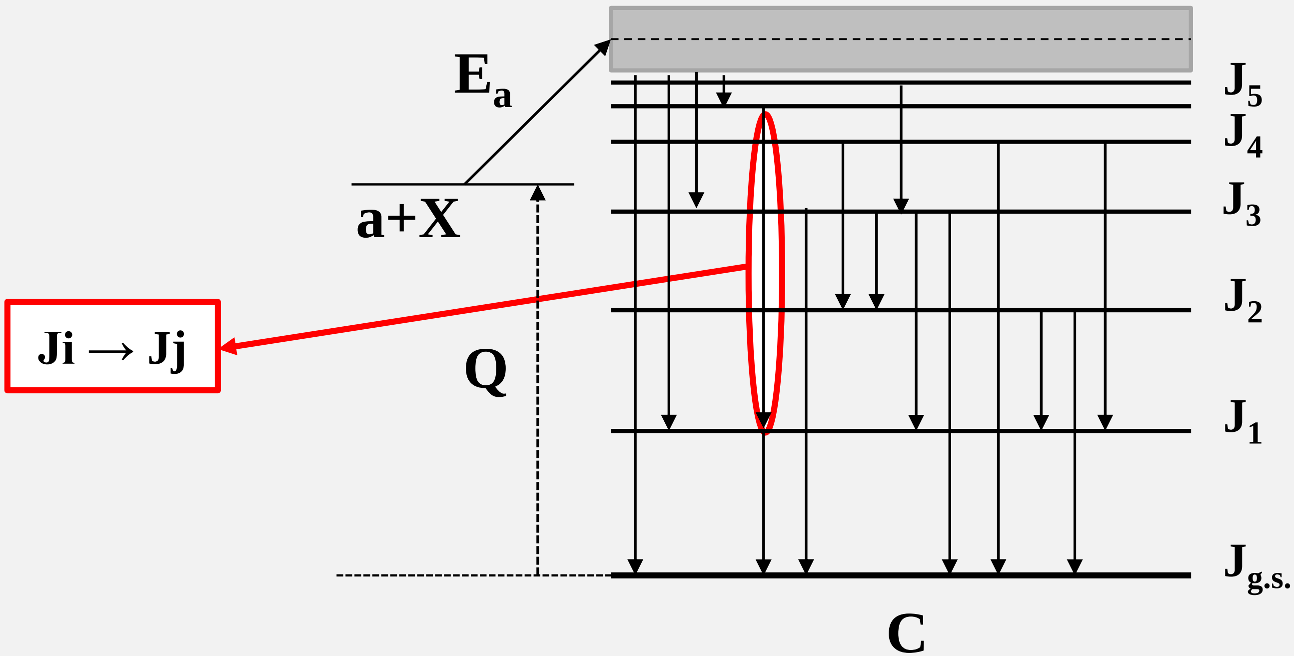
- Spectrum analysis $\rightarrow \mathbf{I}\,(\mathbf{E}_i^{\gamma}, \boldsymbol{E}_j^{beam}, \boldsymbol{\theta}_{\kappa})$
- Corrections $\omega(\theta)$, $\varepsilon_{abs}(\theta) \rightarrow \mathbf{I'}\,(\mathbf{E}_i^{\gamma}, \boldsymbol{E}_j^{beam}, \boldsymbol{\theta}_{\kappa})$
- Normalization Q $\rightarrow \mathbf{Y}\,(\mathbf{E}_i^{\gamma}, \boldsymbol{E}_j^{beam}, \boldsymbol{\theta}_{\kappa})$
- Fit $\Upsilon(\theta)$ with function:

$$W(\theta) = A_0\left(1 + \sum_k \alpha_k P_k(\cos\theta)\right),$$
$$k = 2, 4, ..$$

Angle integration method*

- **Step 1:** Experimental determination of the differential gamma-production cross section

$$\frac{d\sigma_\gamma}{d\Omega}\left(E_i^{\gamma}, E_j^{beam}, \theta_\kappa\right)$$



*L.C. Mihailescu et al., Nucl. Instr. Meth **A531**, 375 (2004)

Angle integration method*

- **<u>Step 1:</u>** Experimental determination of the differential gamma-production cross section

$$\frac{d\sigma_\gamma}{d\Omega}\left(E_i^\gamma, E_j^{beam}, \theta_\kappa\right)$$

- **<u>Step 2:</u>** Determination of the angle-integrated gamma-ray production cross section

$$\sigma_\gamma\left(E_i^\gamma, E_j^{beam}\right)$$

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Angle integration method*

- **Step 1:** Experimental determination of the differential gamma-production cross section

$$\frac{d\sigma_\gamma}{d\Omega}\left(E_i^\gamma, E_j^{beam}, \theta_\kappa\right)$$

- **Step 2:** Determination of the angle-integrated gamma-ray production cross section

$$\sigma_\gamma\left(E_i^\gamma, E_j^{beam}\right)$$

- **Step 3:** Determination of the reaction cross section for every beam energy

$$\sigma_T\left(E_j^{beam}\right)$$

*L.C. Mihailescu et al., Nucl. Instr. Meth **A531**, 375 (2004)

Angle integration

Step 2:

Differential gamma-production cross section:

$$\frac{d\sigma_\gamma}{d\Omega}(\theta_\gamma) = \frac{\sigma_\gamma}{4\pi}\sum_k c_k P_k(cos\theta_\gamma) \quad , k = 0, 2, 4, \ldots$$

Numerical integration of the differential cross section by the Gaussian quadrature method:

$$\sigma_\gamma = 2\pi \int_{-1}^{1} \frac{d\sigma_\gamma}{d\Omega}(x)dx = 2\pi \sum_{i=1}^{n} w_i \frac{d\sigma_\gamma}{d\Omega}(x_i)$$

x=cosθ, n: number of detectors

Requirement: The sum gives the exact result for the integral → Calculation of the appropriate $\boldsymbol{w_i}$, $\boldsymbol{x_i}$

Determination of w_i, x_i

- $x_i = cos\theta_i$ equals to the root of the Legendre polynomial P_{2n}
- The weight equals to:

Number of detectors	Weight wi	Angle θ	
n = 1	2	55	125
n = 2	1.304	31	149
	0.696	70	110
n = 3	0.936	76	104
	0.722	49	131
	0.343	21	159

Case of 2 detectors:

$$\begin{Bmatrix} w_1 + w_2 = 2 \\ w_1 P_2(x_1) + w_2 P_2(x_2) = 0 \\ w_1 P_4(x_1) + w_2 P_4(x_2) = 0 \end{Bmatrix}$$

Exact approach for polyonomials of degree: $\leq 4n - 2$

Determination of w_i, x_i

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Case of 2 detectors:

110° (or 70°) and 150° (or 30°) $\rightarrow P_4(cos\theta) = 0$

$$\sigma_\gamma = 2\pi\left[w_1 \frac{d\sigma}{d\Omega}(110^o, \mathrm{E}_{beam}) + w_2 \frac{d\sigma}{d\Omega}(150^o, \mathrm{E}_{beam})\right]$$

Reaction cross section

Step 3:

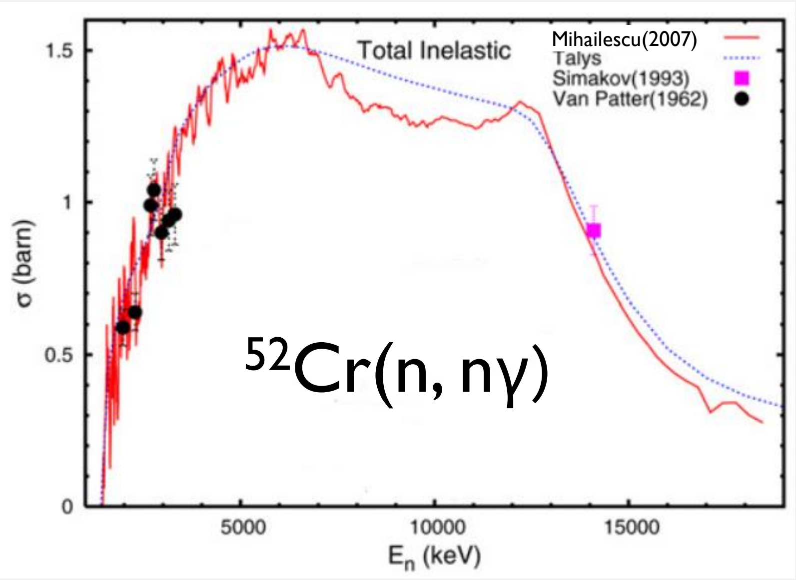
The reaction cross section for each beam energy is calculated using the formula:

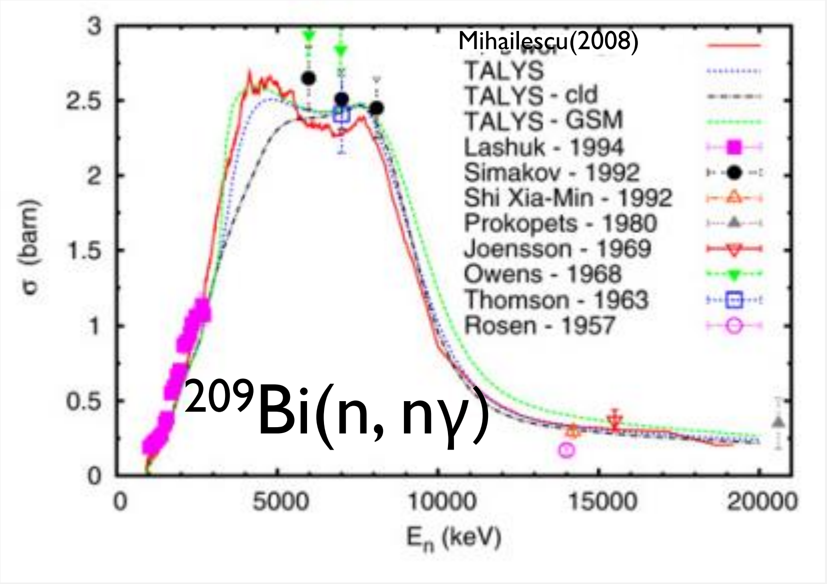
$$\sigma(\mathrm{E}) = \sum_{i=1}^{N} \sigma_\gamma(E^\gamma, \mathrm{L}_i \rightarrow g.s.)$$

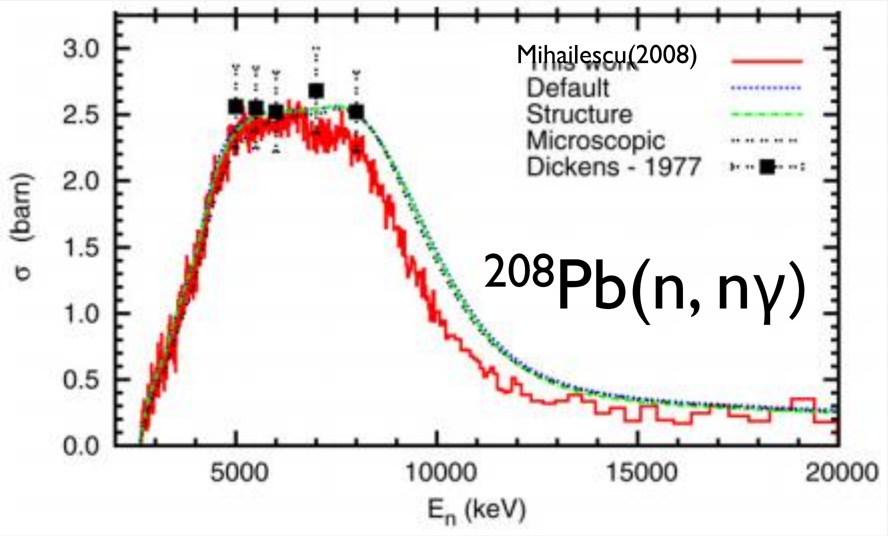
N: number of transitions that lead to the ground state

σ_γ: gamma-ray production cross-section for gamma-ray energy E^γ

Measurements by the angle integration method





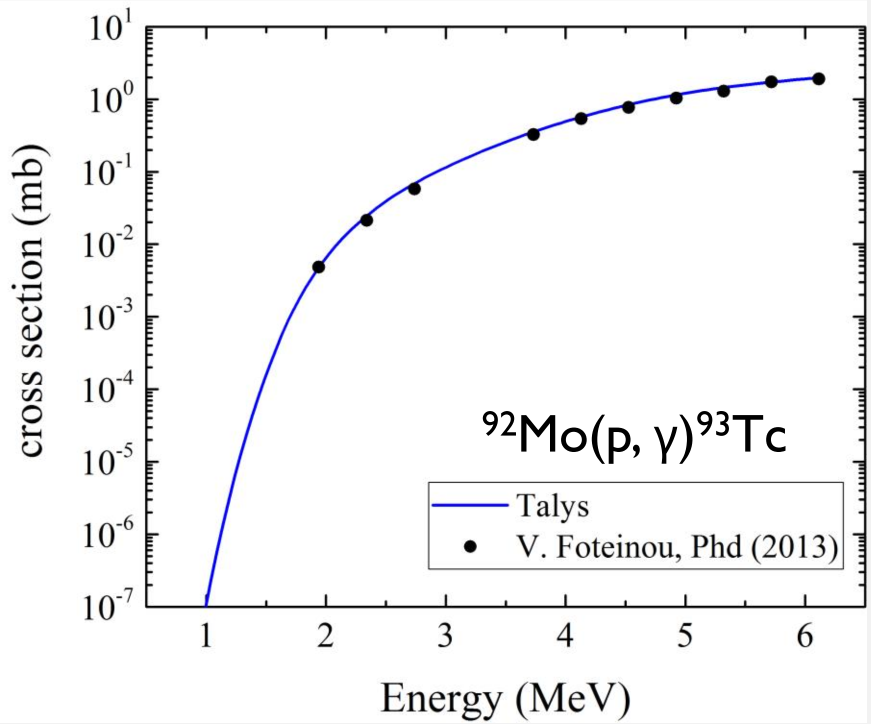


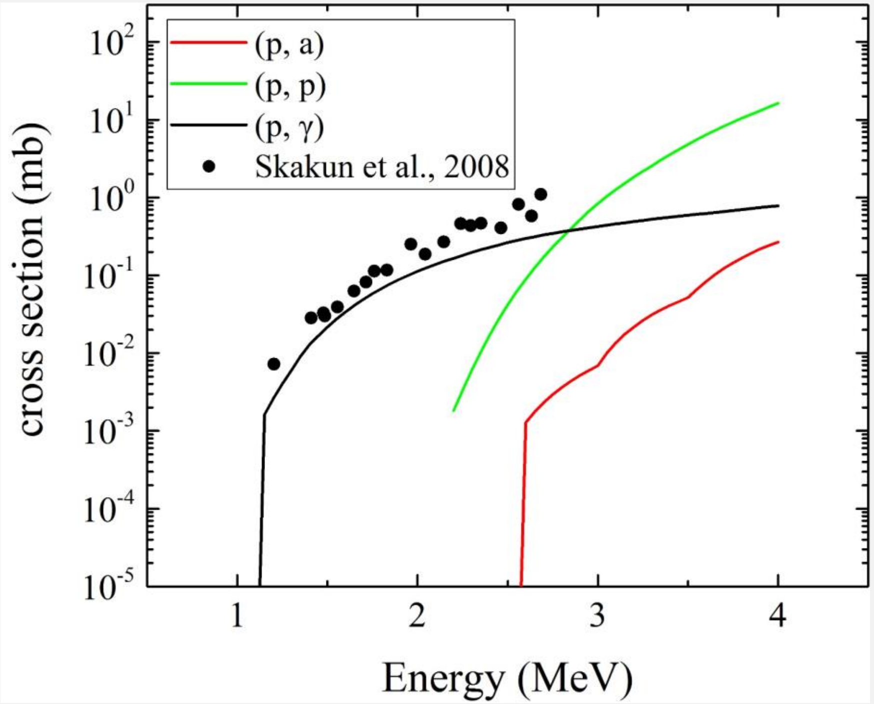
Good agreement of the data with existing experimental data and with TALYS calculations

L.C. Mihailescu et al., Nucl. Phys. ***A799****, 1 (2008)*
L.C. Mihailescu et al., Nucl. Phys. ***A811****, 1 (2008)*
L.C. Mihailescu et al., Nucl. Phys. ***A786****, 1 (2007)*

Future plans

Application of angle integration method to well known reaction





Cross section measurement of proton capture reaction → $^{66}Zn(p,\gamma)^{67}Ga$

Conclusions

- Need for fewer detectors
- Less experimental data to be analysed
- Validation of the angle integration method for the calculation of capture reaction cross section

Thank you for your attention

Number of detectors	$P_{2n} = 0$	Weight wi	Angle θ	
n = 1	0.57735	2	54.74	125.26
n = 2	0.33998	1.30429	30.56	149.44
	0.86114	0.69571	70.12	109.88
n = 3	0.23862	0.93583	76.19	103.81
	0.66121	0.72152	48.61	131.39
	0.93247	0.34265	21.18	158.82

Solution of the equation system

For k = 2

$$\int_{-1}^{1} \frac{d\sigma}{d\Omega}(cos\theta)dcos\theta = \sum_{i=1}^{n} w_i \frac{d\sigma}{d\Omega}(cos\theta_i)$$

Για n = 2:

- $\frac{d\sigma}{d\Omega}(cos\theta_i) = 1 + c_2 P_2(\cos\theta_i) + c_4 P_4(\cos\theta_i)$
- $\int_{-1}^{1} \frac{d\sigma}{d\Omega}(cos\theta)dcos\theta = w_1 \frac{d\sigma}{d\Omega}(\cos\theta_1) + w_2 \frac{d\sigma}{d\Omega}(\cos\theta_2)$
- $\int_{-1}^{1} \frac{d\sigma}{d\Omega}(cos\theta)dcos\theta = \int_{-1}^{1}\{1 + c_2 P_2(\cos\theta_i) + c_4 P_4(\cos\theta_i)\}dcos\theta = 2 + 0 + 0$
- $w_1(1 + c_2 P_2(\cos\theta_1) + c_4 P_4(\cos\theta_1)) + w_2(1 + c_2 P_2(\cos\theta_2) + c_4 P_4(\cos\theta_2)) = 2 \Rightarrow$

$$\left\{\begin{matrix} w_1 + w_2 = 2 \\ w_1 P_2(\cos\theta_1) + w_2 P_2(\cos\theta_2) = 0 \\ w_1 P_4(\cos\theta_1) + w_2 P_4(\cos\theta_2) = 0 \end{matrix}\right\}$$